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differing by some 10 per cent. Considering the whole period, however, the application of the method would have been distinctly helpful. A more discriminating use of the method might easily have been made to yield better results.

Recognition of the fact that various series have various characteristics with respect to secular trend and that these characteristics should be taken into account in its measurement is one safeguard in measuring secular trend. Another safeguard is the recognition of the facts that trend may be affected either by the accumulation of small influences or by a catastrophe of some kind. In other words, we may not always conclude that the trend of the future will be the same as the trend of the past. The considerations upon which an estimate of trend is based are identical with those at the basis of all plans for the future. The basis of such plans is past experience interpreted in the light of careful observation of current developments. To refuse to estimate or plan for the future because our estimates may be in error is to deny the value of experience and the systematic character of human affairs. With past data available an inexact estimate of secular trend is better than no estimate at all.[1]

We may now set forth our conclusions regarding the different methods that have been examined. The method of curve-fitting is superior to the method of moving averages for measuring secular trend. The determination of a curve or line which pictures the secular trend of a past period, does not determine present or future trend. The presumption that past trend will continue is strong in some cases and weak in others. The estimate of future trend should be influenced by recent tendencies and current items to some degree, yet we should not lightly conclude from short-time fluctuations that secular trend has changed. In a period like 1903–18, for instance, it seems best not to include the war years 1917 and 1918 in fixing the line of secular trend for the present and immediate future. For prices it seems best not to include any of the years 1915 to 1918, during which the price level of the world has risen so enormously. The extension of a past trend is a prophecy. It is impossible to get away from that fact. The important thing is that the exact nature of the prophecy be made unmistakable.

In the application of the method to data for the period 1903–16 all the years were assigned equal weights. A complete explanation is given in Part II in connection with the analyses of individual series.

3. Measurement of Seasonal Fluctuations

The object of the present study of seasonal fluctuations is, first, to determine the existence or non-existence of such fluctuations in various series of monthly data; second, if fluctuations exist, to measure them; and third, to correct the items for seasonal movement. By seasonal movement is meant a consistent variation from one month to the next. Are the items for certain months of the year systematically or regularly different from the items for other months ? If so, there is a seasonal variation.[2]

A consistent variation in a series of monthly totals may be due either (*a*) to a difference in the number of days in the adjacent months, or (*b*) to a difference due to greater activity during exactly equal intervals of time, or to both combined. A variation due to (*a*) may be called an artificial variation, and one due to (*b*) may be called a real seasonal variation. In case we are dealing with items that are not totals, such as prices or interest rates, a consistent variation would not, of course, be due to a difference in the lengths of the months.

A variation in totals due to a difference in the lengths of the months might be corrected by taking daily averages. Certain difficulties, however, would be encountered in getting a correct average. Should we include Sundays or merely take week days ? Should we allow for holidays ? These questions would have to be settled not only for each series but for each year. Moreover, it is not true that the statistical items always refer to calendar months. A " statistical month " ending with the 25th or some other fixed date may have been used. In such a case February would contain 31 days and March 28. In order to make any reliable correction for the varying lengths of the months, the exact usage for each series over a long period of time would have to be ascertained; an error of one day would make a difference of three per cent in the monthly total. In consequence of these uncertainties and the laborious computation required to secure correct daily averages, it was decided to take the data as they are presented and to measure the seasonal variation due to all considerations combined.

That many series of business statistics consisting of monthly or quarterly items present marked seasonal variations is recognized both by producers and by consumers of such statistics. That such seasonal variations must be taken into account if we are to use the data as indices of business conditions is also recognized. The method commonly used by periodicals publishing data is to express the item for the current month as a percentage of the item for the corresponding month of the previous year. This simple device has the defect that the basis of comparison is but one item; and that item may be exceptional in character. In order to avoid the difficulty just stated another method has been suggested. "For each subject let a monthly index number be obtained by dividing the actual figure for the month by the average for that month during the ten preceding years."[1] It is claimed that "by using the ten-year monthly averages, seasonal fluctuations are automatically allowed for."[2]

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[1] In presenting its Production Index the Federal Reserve Board takes the position that it is useless to attempt to measure secular trend. The *Federal Reserve Bulletin* of December 1918 (p. 1194) contains the following paragraph:

" Objection may be raised to the above method on the ground that no allowance is made for growth — in technical language, that the secular trend is not eliminated. A comparison with conditions existing prior to the outbreak of the war is alone afforded. But reflection will show that this is really no defect at all. For the methods which have been developed for the elimination of the secular trend are all based upon the theory of normal growth, and require that when disturbing factors cease to operate this normal growth shall again be resumed. Who, however, can maintain that the conditions of the past few years may not permanently have changed the direction of the curve which shows this normal trend ? This brings to light the other objection to the attempt to eliminate the secular trend; the attempt is based upon past data only, and hence data which are incomplete and insufficient for the solution of the problem, especially at a time of great change. All that can be done at present is to contrast conditions now prevailing with those prevailing prior to the outbreak of the great war, and thus indicate in their totality the changes which have occurred."

[2] The week appears to be too short an interval for the measurement of seasonal variation. It is not always clear what are corresponding weeks of a series of years. Days are certainly too short intervals to deal with although in some cases it may be profitable to use weekly items. In general it may be said, however, that if a weekly seasonal variation is so evanescent that it does not show in the monthly averages, such variation is not worth while considering for practical purposes. We must be just as careful not to choose too short a period for measuring seasonal variation as to choose too long a period. The month has been adopted here as the most satisfactory unit.

The method just described and other methods that have been suggested, are not satisfactory for our purpose because, first, they assume that a seasonal movement exists for all series without establishing the fact; and second, if the movement does exist, these methods afford no means of measuring it. No attempts have been made to establish the systematic character of the fluctuations other than the striking of the arithmetic average of the items for the same months for a number of years. This average is affected by cycles of prosperity and depression and by irregular influences, as well as by the true seasonal element, if present. Exceptional items may give an average that is spurious in so far as seasonal variation is concerned.[3]

Although we wish to ascertain if a systematic variation exists it is not accurate to think of seasonal variation (or, for that matter, the other types of fluctuations) as being *exactly* the same year after year. To illustrate: there is admittedly a seasonal variation in temperature but there is a wide dispersion in the percentage changes from, say, September to October and October to November of various years. It is quite probable that a given September, or a small number of months, will be exceptional. It is not safe, therefore, to make a conclusion as to normal seasonal variation from one case or from three or four cases. Here, as elsewhere, the more numerous the data the more certain will be conclusions drawn from them. A considerable amount of evidence is required to establish system in the midst of variation.[4]

Our first task, therefore, was to examine various series with the object of discovering if a systematic movement occurs within the year. The problem was approached by the use of the graphic method. Monthly data, 1903–16, for the following series were plotted in such a way that items for the same month fell in the same vertical column.

Bank clearings of New York City.
Bank clearings of the United States outside New York City.
Building permits issued in twenty leading cities.
Production of pig iron in the United States.

As relative, rather than absolute, changes are significant, the ordinates were plotted on the logarithmic scale, by which equal vertical distances represent equal percentage changes in one direction. Charts D and E present the graphs for New York clearings and outside clearings, respectively. It has not been thought necessary to reproduce the charts for the other series, since they add nothing to the conclusions drawn from Charts D and E. Chart D shows the widely different percent-

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[1] "Statistical Indices of Business Conditions," by M. T. Copeland, *Quarterly Journal of Economics*, 29, p. 554.

[2] *Ibid.*, p. 556. See the same article pp. 543–554 for criticisms of the methods of the Babson Statistical Organization and the Brookmire Economic Chart Co.

[3] The two methods used by E. W. Kemmerer in his study of *Seasonal Variations in the Relative Demand for Money and Capital in the United States* (Report of the National Monetary Commission, 1910) depend upon the assumption that non-seasonal elements will cancel each other when averages are taken. He says, "Throughout the report the various kinds of money-market data are combined in two ways: First, a simple average of the figures themselves; and second, a simple average of index numbers. The first way may be illustrated by reference to the figures for call rates of interest in New York City. The average rate for each week of the nineteen years, 1890–1908, was taken, and each year was adjusted so as to conform to the nearest fifty-two complete weeks. The rates for the first weeks of the nineteen years (1890–1908) were then averaged together, then the rates for second weeks, and so on through the fifty-two weeks of the year. The resulting composite is taken as evidence of the extent of seasonal variations during what is considered a typical year for the period.

"In computing index numbers the following method was employed: Each year was taken as a unit; the average rate for the lowest week in each year was designated by an index number of 0, the average rate for the highest week in the same year by an index number of 100, and the average rate for each of the other fifty weeks was pro-rated. The weekly index numbers so obtained for each of the nineteen years were combined by averaging together those of the nineteen first weeks, then those of the nineteen second weeks, and so on through the fifty-two weeks of the year. The resulting composite is treated as evidence of the regularity of seasonal variations during the period, or, more correctly, of the degree to which the low rates, whatever they are, tend to cumulate at certain seasons, to which the high rates tend to cumulate at certain seasons, to which the high rates tend to cumulate at certain other seasons, etc. If the maximum weekly rate for the year should in every year occur in the same week, the average index number for that week obviously would be 100, and if the minimum weekly rate in every year should occur in the same week, the average index number for that week would be 0."

The first method appears to me to be better than the second, as the yearly scales in the latter are determined by the extreme items. The items *are* extreme not because of the seasonal element but because of the cyclical and irregular elements. With these non-seasonal scales, I have no confidence in the accuracy of the measurements secured.

[4] The Federal Reserve Board holds that (*a*) the use of an average for the corresponding months of three years as a base suffices to remove seasonal variation, and (*b*) the expression of the item for any month as a percentage of the arithmetic means of figures for an entire three-year period gives a measure of seasonal variation. The following paragraph is taken from the *Federal Reserve Bulletin* of December 1918 (p. 1194):

"Current figures will be expressed as percentages of the averages of figures for the same periods in each of the years 1911, 1912, and 1913. An average will be struck, for example, of the amount of grain received at the more important seaboard points during the months of October 1911, October 1912, and October 1913, and the figure for October 1918 expressed as a percentage of the average receipts during the earlier months. By the use of this method allowance will be made for seasonal variations in the amount of grain received. But it is desirable for certain purposes to show the actual seasonal variation as well. Hence current figures will also be expressed as percentages of the averages of figures for the entire three-year period. To employ the previous illustration, an average will also be struck of the amount of grain received monthly at the more important seaboard points during the years 1911, 1912, and 1913, and the grain receipts during October 1918, expressed as a percentage thereof."

In the work which follows in this REVIEW monthly data for a period of from 14 to 26 years were used to measure seasonal variation. I consider 10 years as brief a period as I would care to use and 25 years is desirable.

CHART D

MONTHLY BANK CLEARINGS OF NEW YORK CITY IN BILLIONS OF DOLLARS, 1903-1916

Drawn on logarithmic scale vertically and arranged so that items for the corresponding months of the 14 years fall in columns

Equal vertical distances between points indicate equal percentages of increase or of decrease (rather than equal increments as in the natural scale)

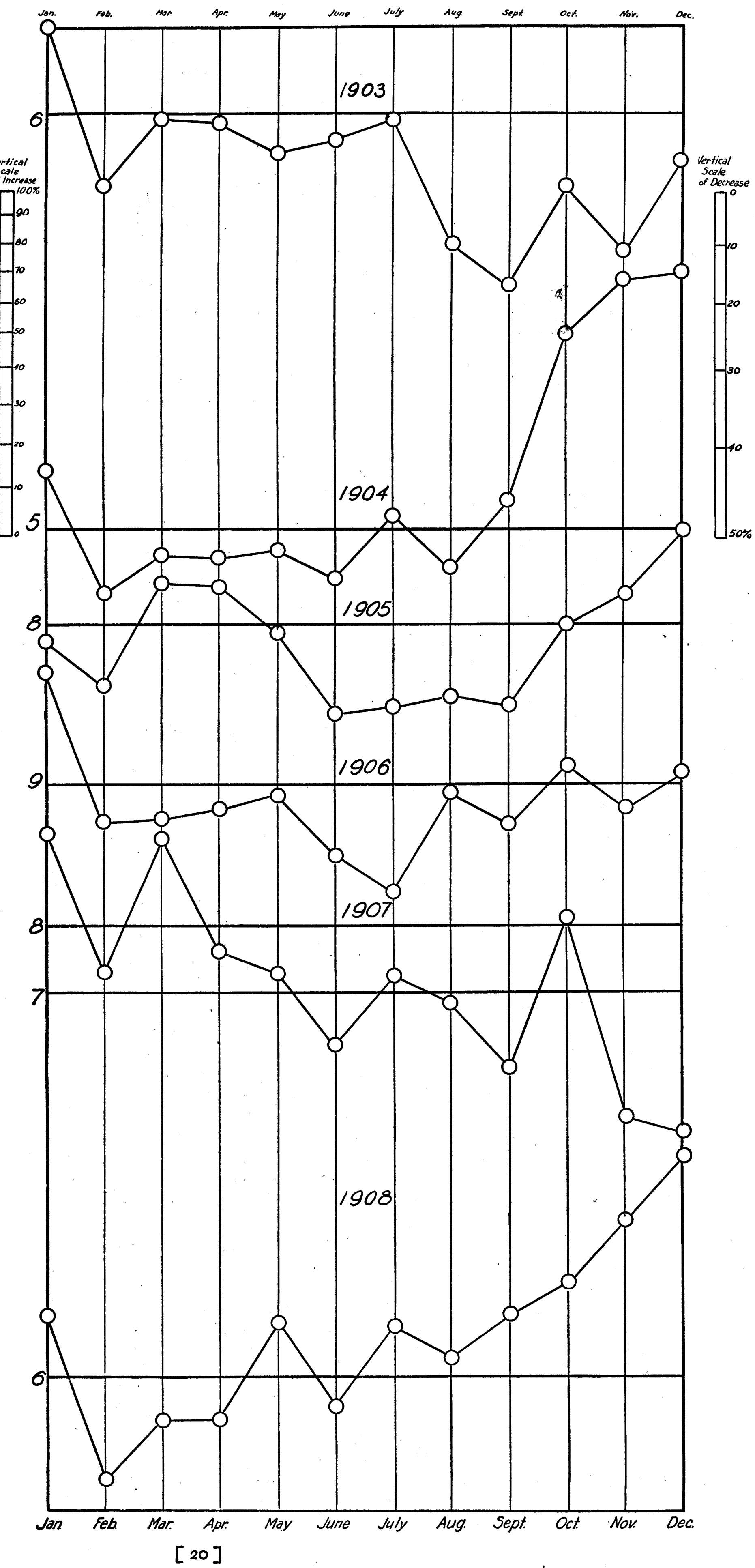


CHART D (CONTINUED)

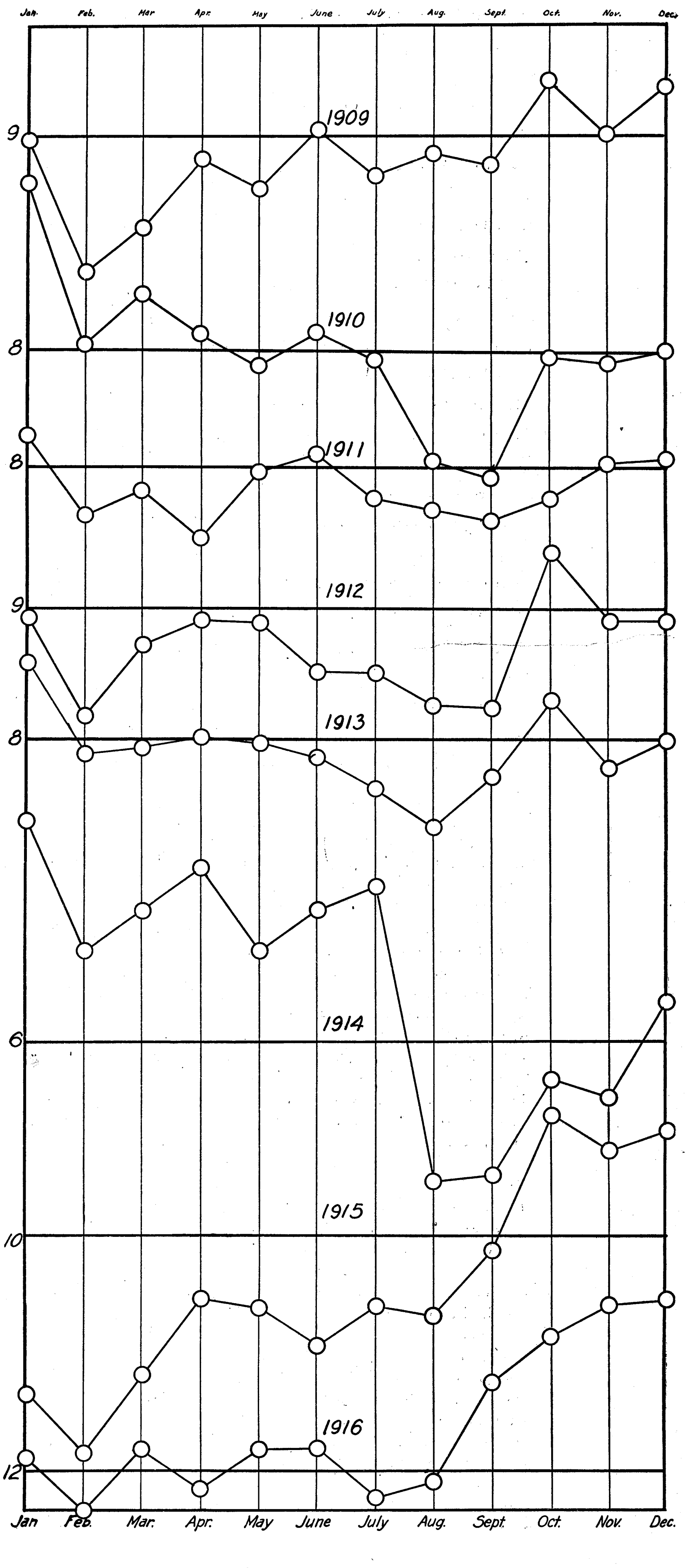
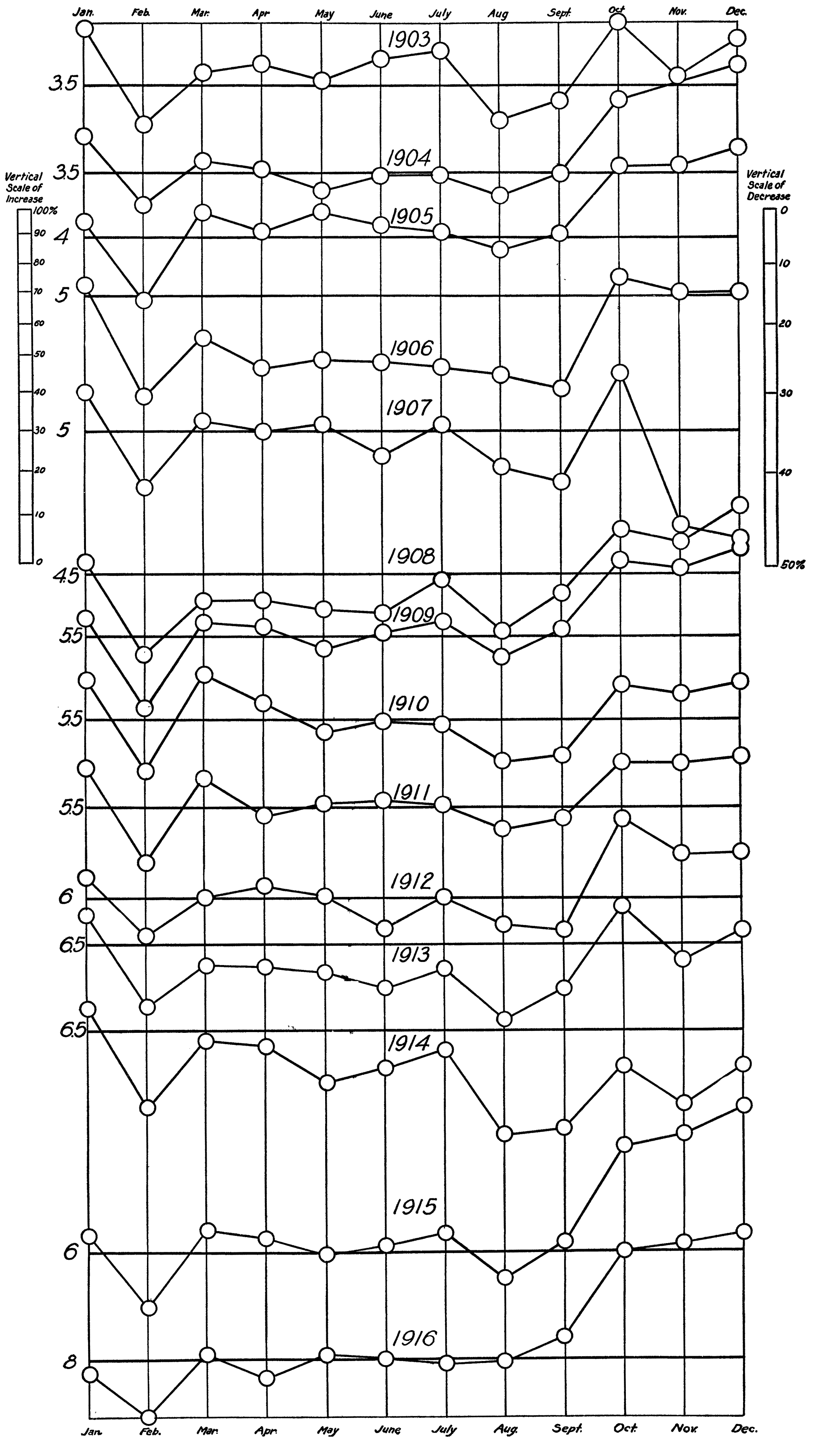


CHART E

MONTHLY BANK CLEARINGS OF THE UNITED STATES OUTSIDE NEW YORK CITY IN BILLIONS OF DOLLARS, 1903-1916

Drawn on logarithmic scale vertically and arranged so that items for the corresponding months of the 14 years fall in columns
Equal vertical distances between points indicate equal percentages of increase or of decrease (rather than equal increments as in the natural scale)



age movement in different years. For instance, compare 1903 and 1904; 1907, 1908, and 1909; 1913, 1914, and 1915. In some cases the autumn months are far below the spring months, in other cases far above. The same situation is revealed by the graphs of other series. Among all the series examined the least diverse movement is shown by outside clearings (Chart E). Yet even in this case there is a considerable divergence in movement in different years. Compare, for instance, the years 1906, 1907, 1908, and 1909. In all of the series, then, it is obvious that the general nature of the graph secured for each year depends, not upon the seasons, but upon the position occupied by the year in the business cycle. For instance, the earlier months of the years 1904 and 1908 were depressed; the later months were months of revival. None of the graphs for the series examined showed a characteristic shape indicative of seasonal variation. If there is a seasonal movement in these series it is effectively covered by the cyclical movement of prosperity and depression. Is it possible to disentangle the seasonal movement from the cyclical movement ?

Up to this point our attention has been directed to the yearly graphs. In comparing the position of the items for the autumn with those for the winter or spring, or with the average for the year, we have, in effect, been using a fixed base. The use of a fixed base allows the cumulative effects of the business cycle to cover the seasonal fluctuation, if there is one. Let us, therefore, concentrate our attention on the month-to-month changes. For New York clearings (Chart D) every February shows a drop from January; every March shows a rise from February; six Aprils show a negligible change from the preceding March, four show a fall and four show a rise; October shows a rise over September in every instance; December is less than November in only one case; the other months show more or less consistent results when compared with the month preceding. The month-to-month movements of outside clearings (Chart E) are still more systematic. In other words, the month-to-month comparison reveals evidence of a seasonal fluctuation. We conclude, therefore, that a study of seasonal variation should be based upon percentages in which the preceding month is the base. These percentages are termed link relatives.

In accordance with the conclusion just stated relatives were formed for various series by dividing the item for each month by that for the preceding month. The resulting quotients, then, give the month-to-month changes. Presumably these changes are due, in part, to irregular and cyclical variations. Nevertheless, a persistent seasonal element should make itself evident. Frequency tables of the relatives were formed, one for each month, twelve for each series. The frequency tables for fourteen series are given in Part II. Examination of these tables shows that the link relatives for any month are grouped, the degree of concentration depending on the series. The relatives for any selected month have, in general, a characteristic " zone of distribution." If for any series there are zones of distribution other than those centering at 100 per cent, we conclude that there is seasonal variation. If the relatives are widely scattered we conclude that there is no seasonal variation, or, if there be one, that it is not significant as compared with other fluctuations.

Let us consider the frequency tables of link relatives for selected series. Table 1 shows that there is a seasonal movement in New York bank clearings. In that table erratic relatives of 56, 67, 130, and 140 per cent are noticeable. By referring to the table of link relatives under Chart 1 we find that such numbers occurred respectively in August 1914, November 1907, March 1907, and October 1904. All of these months fall in years of disturbance, and therefore the relatives are not indicative of seasonal variation.

Table 3 reveals a very systematic seasonal variation in outside bank clearings. In that table erratic relatives of 114, and 75, and 97 per cent occurred in January 1908, and November 1907, and December 1907. The low relatives in November and December occurred in the months following the panic which broke in October 1907; the high relative in January 1908 is due primarily to the low December base. In other words, the extreme items are extreme not because of seasonal variation but because of the financial panic and business crisis of 1907.

Table 4 shows that *Bradstreet's* average prices of commodities are not affected by marked seasonal variation, as the relatives are grouped closely about 100 per cent. The seasonal fluctuation is less than one per cent, and we should have, therefore, to measure variations in units at least as small as one-tenth of one per cent. The questionable accuracy of the series to one-tenth of one per cent, and the violence of price fluctuations due to other than seasonal influences do not warrant us in attaching significance to such measurements. In this series erratic relatives of 96, 106, 112, 95, 96, 106 and 107 per cent occurred respectively in January 1914, February 1915, September 1914, October 1914, November 1914, November 1916, December 1916. Except the item for January 1914 all of the erratic relatives occurred during the periods of depression and revival following the declaration of war in July 1914. They measure, therefore, not seasonal variation but something quite different.

Table 8 for volume of shares traded on the New York Stock Exchange is of quite a different type from Table 4 for commodity prices. The relatives are widely scattered in every month, ranging from less than 70 to more than 140 per cent in nearly every month. The conclusion is clear that, if the month is the unit considered, there is no systematic seasonal variation in this series.

In the case of the value of building permits, Table 6, the relatives with class intervals of one per cent are apparently grouped in zones of distribution. Such zones become more clearly marked if the class intervals are widened to 2 per cent, as they are in Table A.

Examination of the other frequency tables on pp. 50–63 reveals the same phenomenon, a characteristic

Table A Monthly Values of Building Permits Issued for Twenty Leading Cities

(Class intervals of 2%: data for January 1903 to January 1917)

FREQUENCY TABLES, ARRANGED BY MONTHS, OF RELATIVES FOUND BY EXPRESSING THE ITEM FOR EACH MONTH AS A PERCENTAGE OF THE ITEM FOR THE PRECEDING MONTH

Lower Limits of Class Intervals

RELATIVES	Jan. Dec.	Feb. Jan.	Mar. Feb.	Apr. Mar.	May Apr.	June May	July June	Aug. July	Sept. Aug.	Oct. Sept.	Nov. Oct.	Dec. Nov.
Below 60								/				
60												
61											/	
63												
65												
67	//							/				
69											/	
71									//			
73	/								/			
75						/					/	/
77	/				/							/
79	/	//		/				//				
81	/						/			/	/	
83					///		///				/	/
85					/		/	/		/		//
87						/	/		/	//	/	/
89	/			/		/	/	/				/
91	/	//			//		///		/	/	/	
93	/	/			///	//		//		/	///	/
95	/			/					/		/	
97						///	//	//	//			/
99									///		/	
101		/		//			/		/	//	//	
103					/	//				/		/
105	//	//		/	//			/				/
107		/		/				/	/			
109												/
111								/	/	//		
113	/	//										
115				/								
117				/		/						
119	/	//			/	/						
121												/
123		/										
125				//				/				
127												
129										//		
131				//		//						
133												
135												
137							/					
139			/									
141										/		
143			/									
145												
147												/
149												
151				/								
153			/									
155			//									
157												
159			/									
161												
163												
165			/									
167												
169												
171			/									
173			/									
175												
177												
179			//									
181												
183												
185												
187			/									
189												
191												
193												
195												
197												
199												
Over 200			//									
Medians	90½	105	168½	111½	92½	97	90½	93½	97	101	92	91½

ADJUSTED MONTHLY INDICES OF SEASONAL VARIATION

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Indices	70	73	122	135	123	119	106	98	95	95	86	78

TABLE B Monthly Rate of Interest on Four-to-six Months Commercial Paper, 1890-1915.

FREQUENCY TABLES, ARRANGED BY MONTHS, OF RELATIVES FOUND BY EXPRESSING THE ITEM FOR EACH MONTH AS A PERCENTAGE OF THE ITEM FOR JANUARY OF THAT YEAR.

RELATIVES	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Below 70					//	//	/	/				/
70											/	/
71				/					/			
72							/					
73						/						
74			/									
75												
76						/						
77							/					
78				/								
79											/	/
80					/		/					/
81						/						
82		/			/							
83											/	
84				/	/	/	/		/			
85					/			/				
86		/		///	/			/			/	
87		/	/				/					
88		/		/	/	/				/		/
89			//		//	/			/	/		
90			/	/								
91		//					/	/	/			//
92		/	/			//	/			/		
93			/	/		/		///			//	
94					/		///	/				
95		//		/			/					/
96		//	/			//				/	/	
97		///	/	///		///	/	/	/			
98		///	/		/					/		
99		/	//	/	////	/	//	/			//	
100	26	////	//	/		//						/
101		/	/	//	////	//	/	/				/
102			/			/	/	/	/			
103		/		///	/		/	//	/		//	
104		/	/			/	/	/		/		/
105			/				/	/	/			
106			/	//				/	/			
107			/		//				//	///	/	//
108				/	/	/	/		/			
109										/		
110			/				/			/	/	
111									///			
112			/				/	/		//		
113				/				/				
114			/							/	//	/
115		/									/	//
116			/			/			/	/		/
117				/			/		/		//	///
118								/	/	//		
119									/	/		
120								/	//			
121							/	/				
122										/		
123								/				
124												
125									/	/		
126												
127								/		/	//	/
128												
129												
130												
131												
132			/		/				/			
133										/		
134												/
135												
136											/	
137								/				
138					/						/	
139												
140												
Over 140				/		/	/	/	//	//	//	///
Medians	100	97	100	98	99	96½	98	103	111	112	108½	107

Table C Bradstreet's Monthly Index of Commodity Prices

(Class intervals of $\frac{1}{10}$ %; data for January 1903 to January 1917)

FREQUENCY TABLES, ARRANGED BY MONTHS, OF RELATIVES FOUND BY EXPRESSING THE ITEM FOR EACH MONTH AS A PERCENTAGE OF THE ITEM FOR THE PRECEDING MONTH

RELATIVES	Jan. / Dec.	Feb. / Jan.	Mar. / Feb.	Apr. / Mar.	May / Apr.	June / May	July / June	Aug. / July	Sept. / Aug.	Oct. / Sept.	Nov. / Oct.	Dec. / Nov.
Below 97.0	/									/	/	
97.0						/						
97.1												
97.2												
97.3	/											
97.4												
97.5												/
97.6												
97.7												
97.8												
97.9					/							
98.0		/		/								
98.1						/	/					
98.2				/	/	/						
98.3		/	/		/			/				
98.4					/							
98.5				/								
98.6							/					
98.7						/	/					
98.8				//	/			/	/		/	
98.9			/					/	/			/
99.0		/				/						
99.1			/			/						
99.2		/		/		/		/				
99.3			/		/							
99.4	/					/				/		
99.5			/			/	/	/			/	
99.6			/		/				/		/	/
99.7	/	//	/		/					/		
99.8					/						/	
99.9			//	/	//					/		
100.0		/				/	/					/
100.1	/	//	/		/		///		/			
100.2	/				/	/						
100.3									/	/	/	
100.4	/		/				/					
100.5		/						//		//		
100.6	/		/				/	//			/	
100.7	/			/		/	//	/	/			//
100.8						/		/				
100.9		/		/							//	
101.0				/					/		/	/
101.1	/					/			/			
101.2	/			/						/		/
101.3		/					//					
101.4			/					/	/	/		
101.5									/			
101.6												
101.7				/				/		/		/
101.8									/	/		/
101.9	/				/					/		/
102.0		/	/						/			/
102.1										/		
102.2				/								
102.3											/	
102.4												
102.5	/							/		/	/	/
102.6												
102.7												
102.8									/			
102.9												
103.0												
103.1												
103.2				/								
103.3												
103.4												
103.5												
103.6												
103.7												
103.8												
103.9												
104.0											/	
Over 104.0		/							/		/	/
Medians	100.4	100.1	99.8	100.3	99.7	99.3	100.1	100.6	101.0	100.9	100.8	101.1

ADJUSTED MONTHLY INDICES OF SEASONAL VARIATION

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Indices	101.3	101.0	100.5	100.4	99.8	98.7	98.5	98.8	99.5	100.0	100.4	101.2

grouping or scattering of the link relatives of each month. The hypothesis that seasonal variation is a month-to-month variation appears to be fully confirmed by the study of link relatives.[1]

Let us retrace our steps for a moment. In a preceding paragraph it was stated that the use of a fixed base for the items of each year was not a useful procedure in measuring seasonal variation. The effect of using a fixed base each year may be illustrated by the results found in applying the method to the monthly rate of interest on four-to-six months paper. Table 13 gives the frequency distribution of link relatives of this series for the period from January 1890 to December 1915. There is a marked grouping of the relatives for the various months of the year. The erratic relatives [2] all occurred in years of unusual disturbance, such as those of the Baring panic which broke in London in November 1890, the panic of May 1893, the panic of July 1896, the Spanish War of April 1898, the depression of 1904, and the Great War beginning in August 1914. Seasonal variation is thus strongly indicated. Had we used January of each year as the base for all the months of that year, thus securing a new set of percentages, the existence of seasonal variation would not have been evident. Table B gives the frequency tables based on such percentages. There is a fan-like distribution of the tally marks, the greatest scattering occurring in December, the month farthest removed from the fixed base. The comparison for this series of the two methods, that of relatives computed from a fixed base and that of month-to-month, or link, relatives, establishes beyond question the conclusion that to ascertain the existence or non-existence of systematic seasonal variation, link relatives (or a similar device) are necessary.

Having established the fact of seasonal variation, the next step is to measure such variation. Such measurement will be based upon the link relatives. If the arithmetic average of the link relatives be taken, the erratic items, the magnitudes of which are not dependent upon the seasonal element, have an undue influence on the size of such average. Since our data are not numerous, however, we think it best not to neglect entirely the extreme relatives. If we had a large number of items in the frequency table and if these items had a clearly marked class of concentration, the measurement corresponding to that class, the mode, might well be taken as the seasonal relative. As we have but few items the mode is not clearly defined, and the measurement of the middle item of each array, the median, is most satisfactory. The extreme items have no more influence in determining the position of the median than do the less extreme items. Of course, where the items are very closely grouped, the medians, arithmetic averages, geometric means, and modes would approximately coincide. The medians of the link relatives are given in the frequency tables.

Theoretical considerations indicate that the median is preferable to the arithmetic or geometric averages for measuring seasonal variations, but what is the difference in actual practice ? Comparison of the medians, arithmetic averages, and geometric averages of the link relatives for certain series is given in Table D. Since the medians are not known more accurately than to the nearest half of one per cent, the differences between the medians and the other averages are given to the nearest per cent in excess (+) or defect (−) as compared with the medians.

The medians and arithmetic averages of outside bank clearings differ by from −2 to +1 per cent. The arithmetic average for November is lower than the median because of the influence of the low link relative of 75 for November 1907. All the other relatives for that month range between 90 and 103. Clearly it makes a decided difference whether we use the median or the arithmetic average even in this case of close grouping of items (see Table 3). For value of building permits the medians differ from the arithmetic averages by amounts ranging from −4 per cent to +6 per cent. The excess of the arithmetic average for March is due to the link relatives of 207 and 237 occurring in 1903 and 1904, all other relatives for that month being between 139 and 188 (see Table 6). The discrepancies for other months are likewise explained by exceptional items occurring in periods of business disturbance.

The differences between the medians and the geometric averages are also significant. This is illustrated by the computation of the various averages for the production of pig iron and the rate of interest on four-to-six months commercial paper. The geometric and the arithmetic averages of the link relatives of production of pig iron for November and December are four or five per cent below the medians because of the relatives of 73 and 78 for November 1903 and 1907, and 81 and 67 for December 1903 and 1907, respectively. Excluding these exceptional figures, the geometric averages for the two months would have been 95.5 per cent and 101.3 per cent, respectively. In other words, the considerable discrepancy between the median and the geometric average is due, almost entirely, to the effect on the latter of the extreme items occurring in years of business disturbance. For the interest rate on four-to-six months commercial paper the excess of the geometric average in August is caused mainly by the relatives of 130 and 139 for August 1896 and 1914; the defect in November by the relatives of 75 and 72 for November 1893 and 1896.

It is clear from these cases that the use of the arithmetic or the geometric averages gives us significantly different results from those obtained by the use of the median. Moreover, the former averages are less reliable for measuring seasonal variation than the latter because

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[1] It is possible, but not probable, that the choice of a period of two or six weeks rather than a month would have revealed a more systematic variation in some cases.

[2] Such as 131 in Jan. 1896, 115 in Feb. 1895, 82 in Feb. 1896, 134 in March 1898, 124 in April 1898, 86 in April 1904, 78 in May 1895, 117 in June 1893, 82 in June 1898, 139 in Aug. 1914, 130 in Aug. 1896, 72 in Nov. 1896, 75 in Nov. 1893, 122 in Nov. 1890, 75 in Dec. 1914, 82 in Dec. 1896, and 124 in Dec. 1895.

TABLE D — COMPARISON OF MEDIANS, ARITHMETIC AVERAGES, AND GEOMETRIC AVERAGES OF THE LINK RELATIVES OF CERTAIN SERIES

1. MONTHLY BANK CLEARINGS OF THE UNITED STATES OUTSIDE NEW YORK CITY

	Jan. / Dec.	Feb. / Jan.	Mar. / Feb.	Apr. / Mar.	May / Apr.	June / May	July / June	Aug. / July	Sept. / Aug.	Oct. / Sept.	Nov. / Oct.	Dec. / Nov.
Medians	101½	84	113½	98½	98	100½	102	93½	103½	116½	98	103
Arith. Avs.	102.4	85.1	113.6	98.0	98.8	100.0	102.4	93.3	102.8	117.3	95.6	103.2
Differences *	+1	+1	0	0	+1	0	0	0	−1	+1	−2	0
2. MONTHLY VALUES OF BUILDING PERMITS ISSUED FOR TWENTY LEADING CITIES												
Medians	90½	105	168½	111½	92½	97	90½	93½	97	101	92	91½
Arith. Avs.	90.8	102.8	172.2	112.6	93.7	102.8	93.3	91.3	93.2	104.1	87.8	97.5
Differences *	0	−2	+4	+1	+1	+6	+3	−2	−4	+3	−4	+6
3. MONTHLY TONNAGE OF PIG IRON PRODUCED IN THE UNITED STATES												
Medians	100	96	114	98	102½	95	100	103	101	107½	96	102
Arith. Avs.	100.5	99.0	114.6	98.3	101.5	95.3	100.0	103.4	101.5	105.6	92.8	97.5
Geom. Avs.	100.2	98.7	114.6	98.2	101.2	95.2	99.9	103.2	101.3	105.4	92.4	96.8
Differences †	0	+3	+1	0	−1	0	0	0	0	−2	−4	−5
4. MONTHLY RATE OF INTEREST ON FOUR-TO-SIX MONTHS COMMERCIAL PAPER												
Medians	93	97	102	97	98½	99½	103	104	107½	100	98	100½
Arith. Avs.	93.1	96.3	103.8	99.0	97.5	99.3	103.7	107.6	106.3	99.3	95.7	100.2
Geom. Avs.	92.7	96.2	103.5	98.6	97.0	98.9	103.5	107.2	106.2	99.1	95.1	99.5
Differences †	0	−1	+2	+2	−1	−1	+1	+4	−1	−1	−3	−1

* Differences to nearest per cent only.   † Differences from median of more distant average to nearest per cent.

of the influence, clearly shown, of erratic items due to business disturbance.

Having determined upon the medians of the month-to-month relatives as the basis for correcting seasonal variation, the next step is to get the medians into the proper form for correcting continuous series of items for the seasonal element. The process adopted is as follows: The medians of the link relatives were multiplied serially in order to get chain relatives with January as the base.[1] It should be observed that the continuous series of relatives thus obtained is different from the series obtained by expressing each of the original items as a percentage of the item for January of each year and *then* taking medians. To illustrate: for the rate of interest on four-to-six months commercial paper the chain relatives with January as the base are (from data of Table 13): 100, 97, 99, 96, 94½, 94, 97, 101, 108, 108, 106, 107; while the medians obtained from relatives with January of each year as the base are (from data of Table B): 100, 97, 100, 98, 99, 96½, 98, 103, 111, 112, 108½, 107. Chart F presents these two series graphically. The conclusion from this comparison is clear: it is essential that we start the process of measuring seasonal variation with month-to-month relatives and *then* adjust the medians to a fixed base. Reversing the process will introduce significant errors, amounting in this case to four or five per cent.

If, however, the geometric averages of the link relatives had been used (instead of the medians) and these averages had been multiplied together progressively to secure a continuous series of relatives with a fixed base, the result would have been identically the same as that secured by expressing the original items in terms of a fixed base and then taking the geometric averages of these percentages.[2] In other words, it is a matter of indifference if we first take relatives and then geometric averages, or first take geometric averages and then relatives. In still other words, the use of month-to-month relatives and the adjustment of them to a fixed base in measuring seasonal variation would have been entirely without point if we had used the geometric average.

Charts G and H show graphically the continuous or chain series of relatives with January as the base computed from medians and geometric averages, respectively, for two series; viz., production of pig iron and interest rates on four-to-six months paper.[3] The point to be considered in connection with these charts is not the difference between the two graphs (with open and shaded points) on each chart. This difference has been

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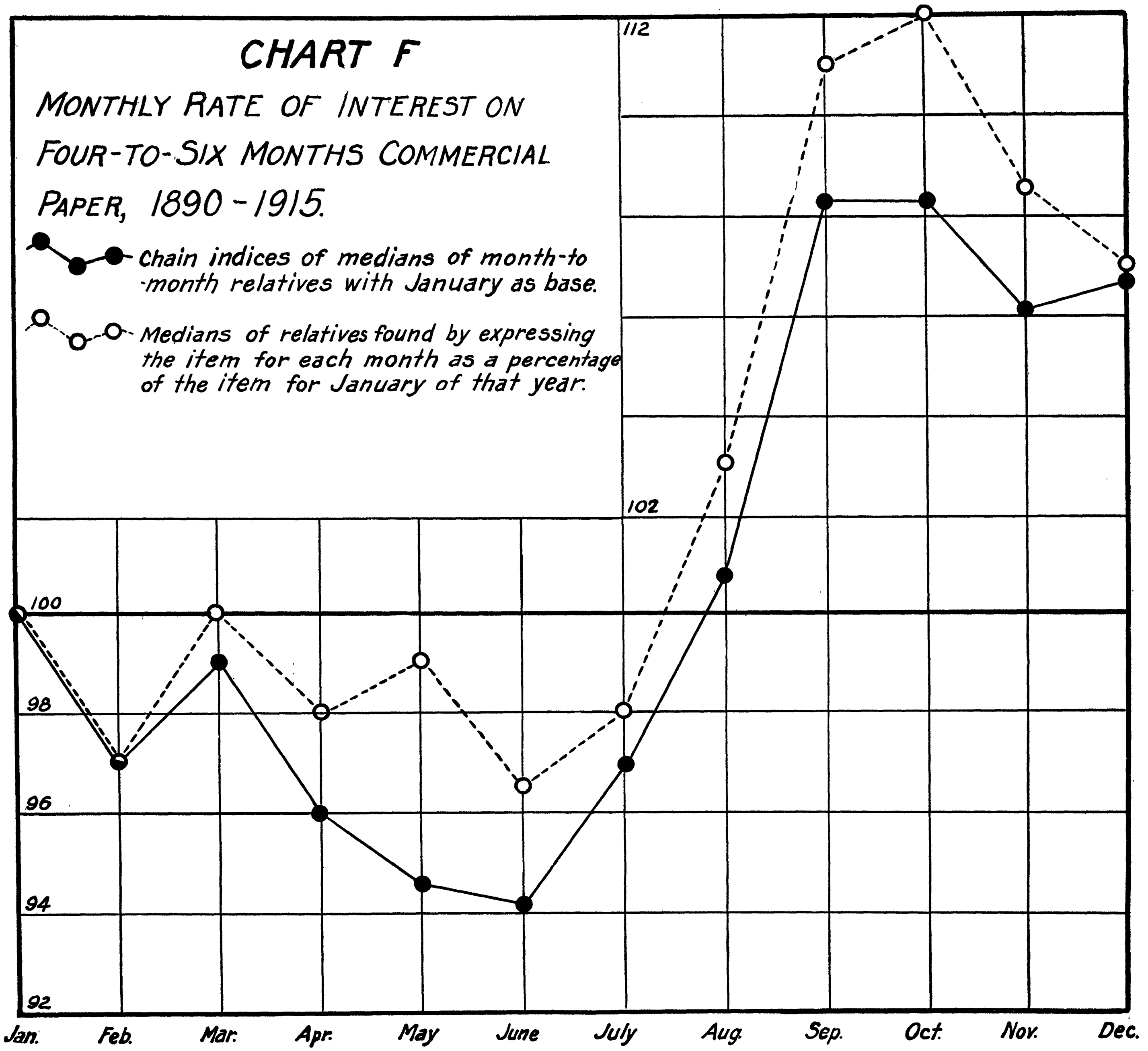
[1] If $r_1, r_2 \ldots r_{11}, r_{12}$, designate the medians of the link relatives or percentages of any series for the twelve months, then 100, $100r_2$, $(100r_2\ r_3)$, $(100r_2\ r_3\ r_4)$, . . . $(100r_2\ r_3\ r_4 \ldots r_{11}\ r_{12})$ constitute the chain relatives with January as the base.

[2] The demonstration of this well known property of the geometric average may be conveniently found in Bowley's *Elements of Statistics*, 2nd ed., p. 223, or Yule's *Introduction to the Theory of Statistics*, p. 124.

[3] Data are from Table D.



discussed in connection with Table D. The point is this: if the chain series be continued after December by multiplying by the January link relative, the resulting relative is not 100 (with which we started, since January was taken as the base), but 114.3 in the case of pig-iron production and 99.3 in the case of interest rates. In the case of other series the discrepancies between the relatives separated by twelve months are considerable, and usually positive. What is the reason for these discrepancies?

The discrepancy between the terminal items, January and January, of the chain series of geometric means is expressible in mathematical terms. It is equal to $\sqrt[n]{\frac{x_{12n}}{x_0}}$ where n is the number of years covered by our monthly series, x_{12n} is the item for December of the nth year, and x_0 is the item for December preceding the first January of the series. In other words, the discrepancy depends upon the ratio between the first and last items in the monthly series.[1] A marked secular trend in the data would, of course, introduce a considerable discrepancy between relatives for January of consecutive years. A discrepancy would be found between other corresponding months were the chain series continued. The discrepancy between the consecutive January items of the chain relatives of the medians is, without question, due primarily to the secular trend of the series. In other words, the figures 100 and 114.3 for pig-iron production and 100 and 99.3 for interest rates are the chain relatives for January items a year apart. In pig-iron

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[1] Proof of this statement may be readily obtained. Let the series of monthly items covering n years be represented by the following:

$$(x_1, x_2, \ldots, x_{12}), \quad (x_{13}, x_{14}, \ldots, x_{24}), \quad (x_{25}, x_{26}, \ldots, x_{36}), \quad \ldots, (x_{12n-11}, x_{12n-10}, \ldots, x_{12n-1}, x_{12n}),$$

where the items in parenthesis are for one calendar year. Let x_0 represent the item for December preceding the item, x_1, for January of the first year of the series. The geometric means of the month-to-month relatives for the twelve months January to December are, respectively:

$$\text{for January,} \quad \sqrt[n]{\frac{x_1}{x_0}\cdot\frac{x_{13}}{x_{12}}\cdot\ \ldots\ \cdot\frac{x_{12n-11}}{x_{12n-12}}}, \qquad (1)$$

$$\text{for February,} \quad \sqrt[n]{\frac{x_2}{x_1}\cdot\frac{x_{14}}{x_{13}}\cdot\ \ldots\ \cdot\frac{x_{12n-10}}{x_{12n-11}}}, \qquad (2)$$

$$\vdots$$

$$\text{for December,} \quad \sqrt[n]{\frac{x_{12}}{x_{11}}\cdot\frac{x_{24}}{x_{23}}\cdot\ \ldots\ \cdot\frac{x_{12n}}{x_{12n-1}}}. \qquad (12)$$

Multiplying the link relatives (1) to (12) inclusive we have

$$\sqrt[n]{\frac{x_{12}}{x_1}\cdot\frac{x_{24}}{x_{13}}\cdot\ \ldots\ \cdot\frac{x_{12n}}{x_{12n-11}}}.$$

Now multiplying the preceding, which is the chain relative for December with January as the base, by expression (1), we have

$$\sqrt[n]{\frac{x_{12n}}{x_0}}.$$

CHART G

MONTHLY RATE OF INTEREST ON FOUR TO SIX MONTHS COMMERCIAL PAPER, IN NEW YORK CITY, 1890–1915

○——○ Chain series of medians of month-to-month link relatives, January as base.

▨----▨ Chain series of geometric means of month-to-month link relatives, January as base.

●—–● Index of seasonal variation or chain series of medians adjusted so that the discrepancy at the end of the year is distributed thruout the year and the average for the year is 100.

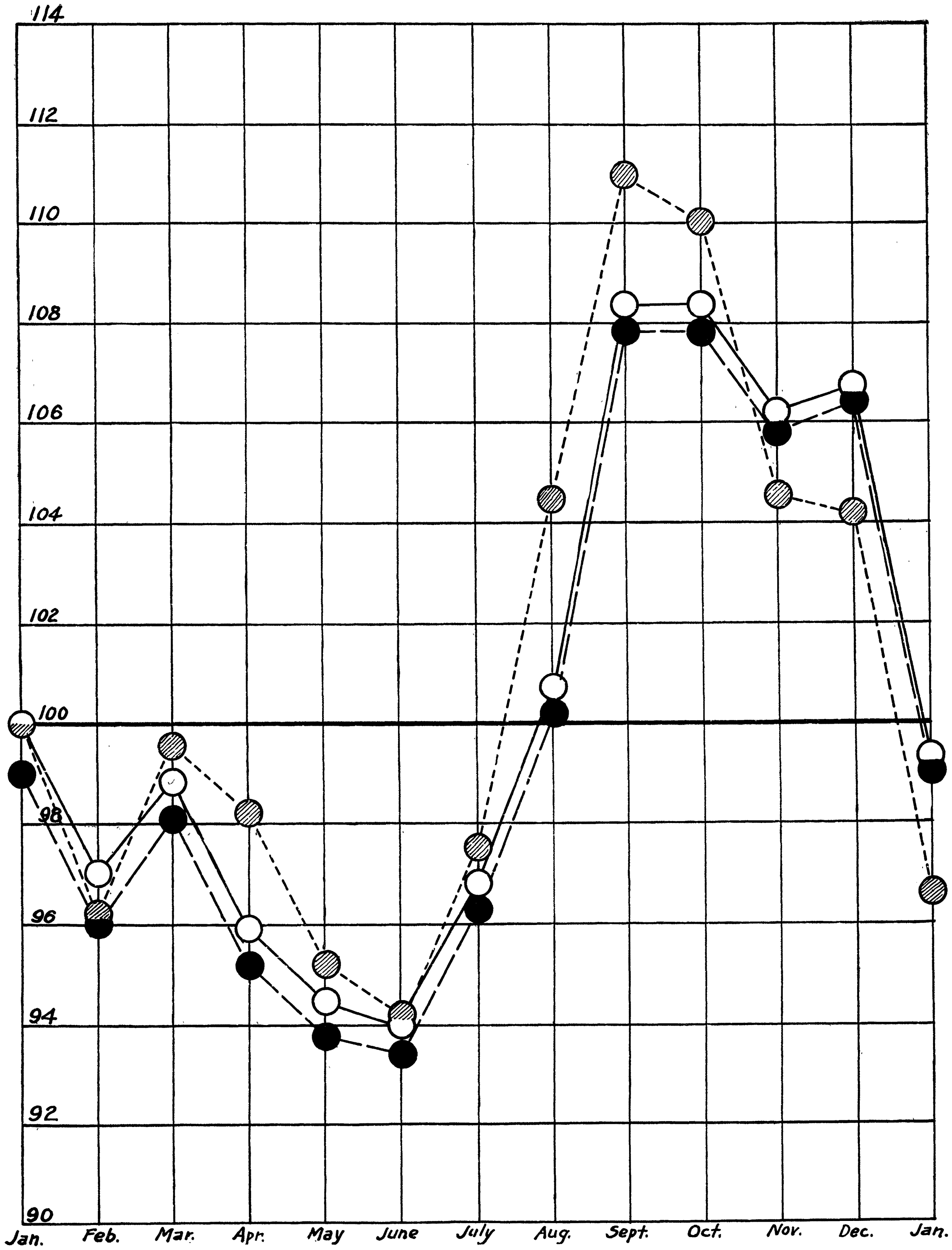

CHART H

MONTHLY TONNAGE OF PIG IRON PRODUCED IN THE UNITED STATES, 1903-1916

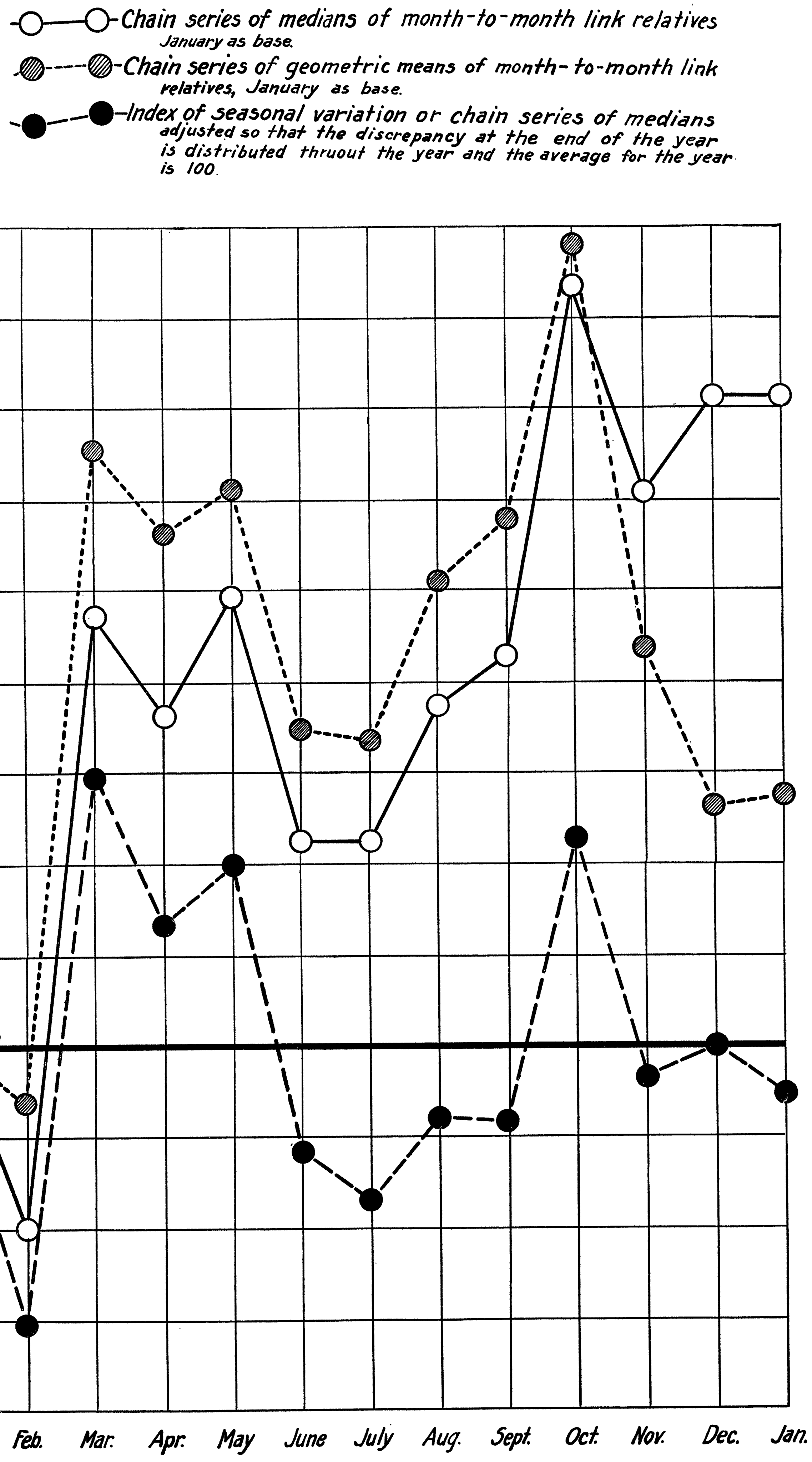

TABLE E — ILLUSTRATION OF METHOD OF SECURING INDICES OF SEASONAL VARIATION FROM MEDIANS OF MONTH-TO-MONTH LINK RELATIVES *

PIG-IRON PRODUCTION

	Jan. / Dec.	Feb. / Jan.	Mar. / Feb.	Apr. / Mar.	May / Apr.	June / May	July / June	Aug. / July	Sept. / Aug.	Oct. / Sept.	Nov. / Oct.	Dec. / Nov.	Jan. / Dec.
Medians of Link Relatives.......	100.0	96.0	114.0	98.0	102.5	95.0	100.0	103.0	101.0	107.5	96.0	102.0	100.0
CHAIN SERIES	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.
With January as 100	100.0	96.0	109.4	107.2	109.9	104.4	104.4	107.6	108.6	116.8	112.1	114.3	114.3
Adjusted for Discrepancy †	100.0	95.0	107.0	103.7	105.1	98.8	97.7	99.5	99.3	105.6	100.2	101.1	100.0
With Arithmetic Average as 100 ‡.	98.9	93.9	105.9	102.6	104.0	97.7	96.6	98.4	98.3	104.5	99.2	100.0	98.9

* Although computations are carried to tenths of a per cent, the original figures and the results are not accurate beyond the units place.

† Adjusted by dividing previous chain series by $(1.011)^{n-1}$ where n is the number of the month and $(1.011)^{12} = 1.143$.

‡ Change to new base secured by dividing previous series by arithmetic average of that series.

production there is a strong upward secular trend; in interest rates a slight downward trend (see Charts 2 and 13.) The discrepancies correspond precisely to these secular trends. In order that the chain relatives may measure seasonal variation only the discrepancy due to secular trend should be distributed; the seasonal relatives should be adjusted to allow for such discrepancy.

The " adjusted monthly indices of seasonal variation " given in Tables 1 to 15 are the original medians of the month-to-month relatives, (1) expressed as a chain series with January as the base, (2) adjusted so that the discrepancy between consecutive January relatives is zero, and (3) changed proportionally so that the arithmetic average of the twelve indices is 100. The effect that the last two operations have upon the chain series of medians is shown graphically for interest rates on four-to-six months paper and pig-iron production in Charts G and H. The graphs with the solid black points represent the adjusted monthly indices of seasonal variation. In these charts the 100 per cent line represents January for two of the graphs and the average for the year in the third. Table E shows the steps in adjusting the indices for pig-iron production.[1] The indices are presented graphically in the inserts of Charts 1 to 15 and these charts are preceded by a summary discussion of seasonal variation for each series.

The reason for expressing the seasonal indices in terms of a fixed base (January or the average for the year) is to secure indices of seasonal variation which may be applied as corrective factors to a continuous series. The items of the original continuous series should be corrected not with respect to the month preceding but with respect to all the months of the year.

That the method of measuring seasonal variation just developed gives consistent results for similar series is evident from Chart K. That chart presents the graphs of the adjusted indices of seasonal variation for the yield on ten railroad bonds, the rate on four-to-six months commercial paper, the rate on sixty-to-ninety days commercial paper, and the rate on call loans at the New York Stock Exchange. In all cases the graphs are of the same general shape but the *extent* of the fluctuations varies greatly. The graphs for the two series most alike — the rates on commercial paper — are most similar both in shape and amplitude. The seasonal (?) variation in the yield of bonds is, of course, minute, in comparison with the call rate. All of the results are in the nature of confirmatory evidence of the reliability of the method developed.

Of the four types of fluctuations which, as has been explained, together make up any series of economic data in its original state, we have now developed methods for measuring two, secular trend and seasonal variation. There remain to be considered the cyclical and irregular fluctuations. Is it possible to measure the element in the original series which is due to the cycle of business prosperity and depression ?

4. MEASUREMENT OF CYCLICAL AND IRREGULAR FLUCTUATIONS

By the term " cyclical fluctuations " is meant that movement of the data which occurs at times of industrial activity or of industrial depression. The movement is called " cyclical " in accordance with accepted terminology. No assumption of fixity or regularity of the cycles is involved in the present use of the term. The requisite is that in order to be cyclical, a succession of large items must be followed by a succession of smaller items, or *vice versa.*

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[1] The method of adjusting the chain indices with January as the base is as follows. (1) To adjust the discrepancy between consecutive Januarys: Let $c_1, c_2, c_3 \ldots, c_{12}$ represent the chain relatives obtained by progressive multiplication of the medians of the link relatives $r_1, r_2, \ldots, r_{11}, r_{12}$. Since January is the base $c_1 = 100$, the product $100 \cdot r_1 c_{12}$ will not, in general, give 100 but something in excess or defect of 100, depending on the direction of the secular trend of the data. The discrepancy between successive January chain relatives is $100 r_1 c_{12} \div 100$, or $r_1 c_{12}$. This discrepancy, due primarily to secular trend, is distributed according to the compound interest law among the chain relatives. Letting the percentage discrepancy for one month be d we have $(1 + d)^{12} = r_1 c_{12}$. Solve for d by the use of logarithms. Then, by logarithms, find the values of:

$$100, \quad \frac{c_2}{1 + d}, \quad \frac{c_5}{(1 + d)^2}, \quad \ldots, \quad \frac{c_{12}}{(1 + d)^{11}},$$

which is the chain series adjusted for the discrepancy. Distribution of the discrepancy on the assumption that it is a uniform monthly increment does not give significantly different results. (2) To change from January as the base to the arithmetic average of the year as the base: Divide each item of the adjusted chain series by the arithmetic average of such chain series for the year.